

Handout: Limits at Infinity and Horizontal Asymptotes

Discussions 201, 203 // 2018-09-19

Problem 1. Compute the limit

$$\lim_{x \rightarrow \infty} \left(\frac{6x^2 + 2}{3x + 7} - \frac{4x + 5}{2} \right).$$

Problem 2. For this problem, we will consider the function

$$f(x) = \frac{e^x - 1}{e^x + 1}.$$

- (1) What is the domain of f ?
- (2) Is f even, odd, or neither?
- (3) Compute $\lim_{x \rightarrow -\infty} f(x)$.
- (4) Compute $\lim_{x \rightarrow \infty} f(x)$. Be careful with limit laws.
- (5) Sketch the graph of $y = f(x)$, keeping in mind your answers to the above questions.

Problem 3. For this problem, we will consider the function

$$g(x) = \frac{7x^2 + 5}{x^2 - 4x + 3} + \frac{\sqrt{x^2 - 4}}{4x + 1}.$$

- (1) What is the domain of g ?
- (2) Find all vertical asymptote(s) of g . Additionally, for each vertical asymptote $x = a$, determine $\lim_{x \rightarrow a^-} g(x)$ and $\lim_{x \rightarrow a^+} g(x)$.
- (3) Compute $\lim_{x \rightarrow -\infty} g(x)$.
- (4) Compute $\lim_{x \rightarrow \infty} g(x)$.
- (5) Sketch the graph of $y = g(x)$, keeping in mind your answers to the above questions.